The Atmospheric Solitary Wave

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ABSTRACT

Solitary waves have been observed in shallow water. In the present paper some evidence is presented for the existence of similar phenomena in the atmosphere. A possible mechanism for the formation of atmospheric solitary waves is described, and a case study is discussed in support of this mechanism. Some speculations are made about some possible effects of these disturbances on the local weather.

INTRODUCTION

It is known through the work of Scott Russell [13] that a particular type of wave, which he called the “solitary” wave, may be formed at the surface of shallow water. A solitary wave is defined in hydrodynamics [7] as a wave which consists of a single elevation of height not necessarily small compared with the depth of the fluid, and which, if properly started, may travel, without change in type, for a considerable distance along a uniform canal. Russell was able to generate solitary waves in the laboratory by careful manipulation of a vertical plate extending across a flume [12].

Some theoretical work has been done by various authors in attempts to explain the formation and behavior of a solitary wave, and to establish the conditions which must be satisfied in order that it may conserve its type. Thus Rayleigh [11] has given derivations of the approximate form of the general profile of a permanent type solitary wave, and established a velocity-amplitude relationship for such a wave. Other investigators, for example, McCowan [9], and Korteweg and DeVries [6], have endeavored to improve Rayleigh’s theory or to arrive at a general theory for periodic waves of finite amplitude, of which the solitary wave may be considered as a special case. More recently, Keller [5] has reexamined the whole problem of periodic waves of finite amplitude formed in shallow water, and has shown that a solitary wave is a special case of periodic waves whose wave length is infinity.

It is known ([7], p. 251) that a finite-amplitude wave of elevation, which obeys the assumptions made in the long wave theory, suffers a continual change of form as it advances, a change which eventually leads to the breaking of the wave. Lamb [7] has, however, shown that a permanent
type wave of elevation is still possible if vertical accelerations are initiated which avert the change of type.

It seems, therefore, reasonable to assume that a wave of a single elevation may either change type and eventually break in conformity with the long wave theory, or it may conserve its type and propagate without necessarily breaking; the behavior of the wave depending upon the vertical accelerations that may exist. Since either kind of waves may exist in the form of a single elevation, it seems permissible to call both of them “solitary waves.”

Munk [10] has been able to show that the breakers which approach a sloping beach conform, very approximately, to the theory of a “permanent” type solitary wave just before they break. His theory was verified by actual measurements. The findings of Munk seem to give support to the idea that the permanent type wave and the breaker are two kinds of the same phenomenon.

The present writer [1] has expressed the conjecture that solitary waves, of similar nature as described in the previous paragraphs, may exist in the atmosphere. A single elevation in the upper surface of a layer of inversion may be created and may travel for a considerable distance along that surface, showing the general appearance of a small migratory high to an observer located on the ground.

Because of the small horizontal dimensions of a solitary wave, in comparison with larger atmospheric disturbances, it has, until recently, escaped notice. With the realization of the fact that some of the small-scale disturbances may produce marked effects in the local weather, and with the development of a dense network of meteorological observatories in this country in connection with severe storms and tornadoes, some of the finer micro-structures of the atmosphere have been revealed. One type of these small-scale disturbances that has been revealed is the micro-anticyclones mentioned by Fawbush and Miller [2] and called by them “the bubbles.” The attention of these authors was drawn to these disturbances because they found them to be associated with tornado formation in certain types of air masses of North America. Fawbush and Miller have surmised that these “bubbles” play the role of the trigger in tornado formation by lifting warm moist air over the cooler air that constitutes them.

The present writer wishes to express the belief that these “bubbles” are actually the “atmospheric solitary waves” which were suggested in his paper of 1949 [1]. The present paper is an attempt to describe these disturbances, to suggest a possible way of their formation, and to show how they may lead to the creation of some convective activity which may even lead to the creation of tornadoes.

A synoptic case in which these disturbances were observed will be discussed in support of the hypothesis made about their nature. The observations for this case study were supplied to the author by Mr. Herman Neustein of the U. S. Weather Bureau to whom the writer is greatly indebted.

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**Fig. 1(a).** The velocity of the plate against time in a possible movement that may give rise to a solitary wave.

**Fig. 1(b).** The movement of the generating plate represented by the displacement against time.
THE FORMATION OF A SOLITARY WAVE

In the laboratory a solitary wave may be created in a rectangular flume, which is partially filled with water, by moving a plate at one end of the flume in such a way that the plate, which is originally at rest, is moved into the fluid to a certain distance, and stopped there. The motion of the plate must be continuous, so that it is given a forward acceleration until it attains a maximum velocity after which it is retarded.

Figure 1a shows the general manner of variation of the plate velocity with time measured from the instant it starts to be accelerated, and Figure 1b is a plot of the plate displacement against time. Figure 2 gives the general form of the resulting wave which from then on travels away from the plate.

![Figure 2](image)

**Fig. 2.** Schematic representation of the single elevation produced by a movement of the nature given in Figure 1.

It is obvious that for the same fluid conditions, the shape of the resulting wave depends mainly on the manner of motion followed by the plate. Moreover, theory indicates that the resulting wave must necessarily change type unless vertical accelerations are produced and are just right to overcome the tendency for this change.

In the atmosphere a similar mechanism may be conceived. Thus a stagnant layer of inversion may be in contact with a stationary front. The stationary front may move into this air because of an impulse it receives from the air behind it, and may stop after advancing for a short distance in some way as shown in Figure 1. A single elevation may then be created in the cold air which may travel as a solitary wave. This wave, which travels like a dome of cold air, will be observed as a small migratory anticyclone. If during the process of its formation the right vertical accelerations are produced, it may travel for a considerable distance without much change in shape except for what may result because of frictional forces and because of the possible changes in the properties of the involved air masses. If the associated vertical accelerations are negligible, the wave will ultimately break and behave like a small pressure jump. We may therefore, as already stated in the introduction, speak of two different kinds of atmospheric solitary waves—the breaking wave and the permanent wave.

AN IDEALIZED EXAMPLE OF THE BREAKING WAVE

The theory of the breaking type of atmospheric solitary waves is the same as that of the pressure jump line which has been extensively discussed by various authors [3, 14]. This theory will, therefore, not be discussed here. It suffices to say that an elevation wave is created in the field when the "plate" accelerates, and a depression wave is formed when the "plate" decelerates. The contributions of the elevation wave and the depression wave which follow it appear as a single elevation in the fluid, if the plate comes to rest while it is moving forward. If the plate is retracted before it comes to rest, the general appearance of the advancing wave would be an elevation followed by a depression in the general undisturbed level of the fluid.

To clarify the process of the formation of these waves the following idealized example is considered.

Let there be a stagnant air mass of depth 2 km and mean temperature 270°C, resting on a horizontal flat earth. The air resting above it is potentially warmer, so that there is an inversion of 4.1°C. In touch with this stagnant air there is a stationary front which starts, at time \( t = 0 \), to move into the air, the motion of the front being given by the following law:

\[
X = A(1 - \cos \omega \, c_0 t),
\]  
(1)

where \( X \) is the displacement of the front measured from its original position, \( A \) is a constant, \( c_0 = \sqrt{g/h} \) is the velocity of an infinitesimal disturbance at the surface of the inversion, \( \omega \) is a constant with dimensions \( 1/L \), and \( t \) is the time measured from the instant the front starts to move. Let \( A \) have the numerical value of 5 km. This means that the maximum distance traveled by the front is 10 km, and it is reached at the instant \( t = \pi/\omega c_0 \). Let \( \omega = 10^{-8} \) cm/s, and from the values assumed previously \( c_0 \) has the value \( 1.75 \times 10^3 \) cm/sec.

From (1) it follows that the velocity of the front at any instant, between \( t = 0 \) and \( t = \pi/\omega c_0 \), may be found from the following equation.

\[
U = \frac{dx}{dt} = \omega \, c_0 \, A \, \sin \omega \, c_0 t.
\]  
(2)
The velocity at time $t = 0$ is zero, and at time $t = \pi$ it is zero again. The maximum velocity $U_{\text{max}}$ is $\omega c_0 A$, which is in this case $0.87 \times 10^4$ cm/sec, attained at the instant $t = \pi / 2 \omega c_0 = 15$ minutes.

Upon neglecting vertical accelerations and hence adopting the assumptions of the long wave theory, the shape of the waves generated on the layer of inversion, by the movement of this front, may be found graphically by the well known method of characteristics [3]. In order to make the graphical solution somewhat more general the following dimensionless quantities are introduced.

$$\begin{align*}
\alpha &= \omega x & \xi &= \omega X & \sigma &= \omega A \\
\eta &= \omega h & \gamma &= \omega \sigma t & \bar{U} &= U c_0^{-1},
\end{align*}$$

where $x$ is the horizontal coordinate of a point in the disturbed fluid.

In terms of these variables equation (1) takes the following form.

$$\xi = a (1 - \cos \tau).$$

The graphical solution for the generated wave is given in Figure 3. It may be noticed that a unit distance in the horizontal direction is equivalent to 10 km in the assumed case, and a unit of $\gamma$ is equivalent to 9.6 minutes. The inserts of Figure 3 give the shapes at various instants. Here the vertical scale is exaggerated 20 times, so that a unit in the vertical represents 2 km, which is the assumed undisturbed depth.

It may be seen from this figure that the wave started to break at the instant $\gamma = 1.6$ ($t = 15.5$ minutes), and at the point $\alpha = 1.8$ ($x = 18$ km). The maximum height of the elevated fluid at this time $\eta_{\text{max}} = 1.52$ ($h = 3.04$ km) above the ground. This represents an elevation of 1.04 km above the undisturbed inversion. The corresponding increase in pressure at the ground is 1.5 mb approximately.

In order to show the subsequent development in the generated wave, the graphical solution was carried out for later instants. This is not strictly legitimate because the method of characteristics
does not hold once a discontinuity is initiated in the flow. However, it has been shown by Friedrichs [4] and Lax [8] that this method yields approximate results in the flow below the jump as long as the jump is weak or moderate.

It may be seen from the insert of Figure 3 that at the instant \( \gamma = \pi \), which is the instant the front reaches its maximum displacement and comes to rest, there is a single elevation whose height is 1.52 (= 3.04 km). The length of the elevated fluid is about 22 km. If the front stopped at its maximum displacement, this elevated fluid would move as a solitary wave which loses strength as it travels because more and more of the elevated fluid would break, until it is finally dissipated.

If the front moves backward, returning to its initial position, the elevated fluid will be followed by a depression as shown in the figure. The general appearance of this case to an observer on the ground would be that of a small high followed by a small low, as may be inferred from the figure.

It seems that, in nature, the stationary front may perform more than one complete oscillation, or it may move in interrupted surges, so that the generated waves will have the form of several elevations separated by depressions. This is probably similar to the case mentioned by Fawbush and Miller when they spoke about the migrating "bubbles."

The Solitary Wave of Permanent Type

The assumptions postulated in the long wave theory lead to waves which change their type continuously, and hence cannot be reduced to a steady case. In order to obtain a wave that may retain its general shape as it travels, vertical accelerations have to be considered, and the general solution must be carried to a higher order of approximation than the solutions obtained by the method of characteristics. The method of characteristics, however, remains a good first approximation to the more exact solution as long as no jump is created.

The author does not wish to discuss the mathematical solutions of these waves in this paper. These solutions will form the thesis of a forthcoming publication. It suffices us to mention here that the process of generation of the solitary wave of permanent type remains fundamentally the same as discussed in the previous section. The stable shape of the permanent type wave is found to be given by the following equation.

\[
\frac{\xi}{h} = 1 + \frac{a}{h} \sech^2 \left[ \frac{\sqrt{3}}{2h} (a/h)^{1/3} x \right].
\]

\[c = \sqrt{g'(h + a)}
\]

where \( \xi \) is the total elevation measured from the ground, \( h \) is the height of the undisturbed inversion layer, \( a \) is the maximum height of the wave above the undisturbed level, and \( x \) is the horizontal coordinate measured with respect to a system of axes which moves with the wave and has its origin on the vertical passing through the peak, the wave being symmetrical around the peak. The same shape is true for water solitary waves [5]. Figure 4 gives the general shape of this type of solitary waves.

These waves travel with a constant velocity which is approximately given by the following equation [7].

\[c = \sqrt{g'(h + a)}
\]

where \( \rho_1 \) is the density of the lower fluid, \( \rho_2 \) that of the upper fluid, and \( g \) is the acceleration of gravity.

Because of the fact that the permanent type waves require a certain particular adjustment between the vertical motion and the horizontal motion, it seems to the present writer that the majority of the cases that are observed in the atmosphere belong to the breaking type. This of course does not mean that the permanent type does not exist, since it is always conceivable that the movements of the fluid adjust themselves in the required manner so that a permanent type may result.

A Case Study

A synoptic situation which was favorable for the formation of atmospheric solitary waves, and
in which a wave of nearly permanent type was created is described in the series of surface maps of June 29, 1951, reproduced in Figure 5. There is a stationary front extending over Colorado, New Mexico and Oklahoma. The cold air to the north of it was rather shallow and nearly stagnant at the surface. The part of it, oriented in the N-S direction, which was extending over Colorado oscillated between 0300C and 0630C. Apparently it received an impulse from the warmer air to the west. This oscillation initiated a wave of single elevation in the cold air which moved eastward. It was first observed in Plevna, Kansas, in the form of a single rise in the pressure shown on the barograph which is reproduced in Figure 6. The peak of this wave passed that station at 0700C. The maximum increase in the pressure during the passage of this wave was 0.1 in. (= 3.4 mb). Its shape, as it moved over Kansas, is indicated by the shape of the rise in pressure at other stations of which two are shown in Figure 6. This wave was not followed later since it left the network of observations. It may be seen from these figures that the general shape of the wave was nearly preserved, and that little or no tendency of breaking was revealed. From the barograph traces the total length of the elevated cold air is estimated to be about 150 km.

Figure 7 gives the isochrones of this particular wave as it passed, and was registered on the barographs, over various stations. It may be seen from this figure that the wave was moving with a fairly constant speed of 87 km/hr at its northern and middle sectors; but it was moving more slowly at its southern sector where its speed was about 65 km/hr. Apparently the cold air was deeper at the northern sectors and it became shallower as the stationary front was approached, as may be expected. Extrapolation of the position

![Figure 5](image1.png)  
**Fig. 5**, a, b, c. Series of surface maps for June 29, 1951, showing the synoptic situation or the date when the solitary wave was observed.

![Figure 6](image2.png)  
**Fig. 6.** The barographic traces at Plevna, Fredonia and Pittsburg, Kansas.
of the isochrones places the wave at station SGR at 1230°C. On the 1230 map a small high was indicated.

The weather on this particular day showed some scattered showers in the region where this wave passed. It is reasonable to assume that these showers were created because of the condensation caused by lifting the air over the solitary wave.

**Some Speculations on the Effect of Solitary Waves on the Local Weather**

Because a solitary wave is an elevated mass of cold air progressing on a layer of inversion, it is expected that it lifts the less dense air which lies above that layer as it passes through it. Its effect may be, therefore, simulated to that of a mountain which forces the wind to rise on its forward slope. Condensation may result because of this lift if the lifted air contains enough humidity so that it may reach its lifting condensation level while it is being raised. If the upper air is stable, the effect of a passing solitary wave would be limited to the formation of a cloud which appears to move with the same velocity as that of the solitary wave. If the wave is of the breaking type the turbulence caused during the process of breaking would result in convective type cumulus clouds; but if it is of the permanent type the clouds tend to be of the stratus form and no turbulence may be expected as long as the air remains stable.

If the upper air is conditionally unstable, so that the lifting caused by the wave may release the instability, the clouds may be expected to develop into the cumulonimbus type, and a thunderstorm may result. In a case of strong instability the convective updraft currents that would result may conceivably lead to the creation of a tornado. The strong vertical accelerations that may be expected in such a case may react on the solitary wave itself and may lead to its destruction.

In order to obtain a rough idea about the height to which the upper air may be lifted, and about the energy released in the process, the example discussed in the previous section may be considered again.

It was found in the previous section that the
increment of pressure rise caused by the passage of the peak is 3.4 mb. Assuming a temperature difference between the two layers of the inversion of 4° C and a mean temperature in the lower layer of 280° A, the height of the peak is found to be nearly 2.5 km above the undisturbed inversion surface. Next assume that such a wave passes in the air mass described by Fawbush and Miller [2] as Type I, and represented by Figure 1 of their paper. In that figure the inversion strength is only 2° C, which would result in nearly double the elevation taken in the present example. The present figure is therefore very conservative.

The inversion layer of Fawbush’s air mass is at the level 820 mb. Its lifting condensation level is at 640 mb. But a lift of 2.5 km raises the air which is initially at the inversion surface to 500 mb. It is therefore more than enough to cause condensation. Actually the convective instability may be released if that air is lifted to 620 mb. A solitary wave of the moderate height postulated here is therefore more than enough to release that instability.

It seems, therefore, plausible to assume that a solitary wave is capable of producing the effects described at the beginning of this section. In particular, if the wave breaks, the turbulence that results in the process supplies some additional mechanical energy and may be more favorable to the initiation of convective instability.

Because of the rather small size of the solitary wave, it is expected that the maximum lifting occurs over a small area surrounding the peak of the wave. Hence the horizontal extent of the convective currents that are produced is expected to be comparatively small. This is believed to help in channelling the updraft within a small cross section, and result in strong vertical velocities at that locality, in addition to the organized pattern of convection that is expected under these conditions. It is surmised that these conditions are favorable for the formation of tornadoes.

REFERENCES